

Fortschr. Phys. **35** (1987) 8-9, 573-673**Zero Trajectories and the Problem of the Reconstruction
of the Phase**

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We discuss the problem of the reconstruction of the phase of the scattering amplitudes of elastic strong interaction processes starting from the observables on extended domains of energy and angle. Analyticity of the amplitudes in one or two complex variables is the main theoretical constraint.

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