On the convergence of reflectionless approximants to confining potentials

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(Received 5 June 1981; accepted for publication 1 August 1981)

We discuss the problem of the convergence of the reflectionless approximants $V_N(x)$, proposed by Quigg, Rosner, and Thacker, to the true confining potential $V(x)$, both for "wrong" $[u'(0) = 0]$ and "correct" $[u(0) = 0]$ boundary conditions. We show that, for any function $\phi(x)$ with continuous derivative and compact support contained in $(0, \infty)$, $\int_0^\infty \phi(x) V_N(x)\, dx \to \int_0^\infty \phi(x) V(x)\, dx$ as $N \to \infty$, if $V(x)$ is appropriately restricted. Namely, for "wrong" boundary conditions, $V(x)$ should be such that the difference $\sigma(E)$ between its spectral measure and the free one obeys $|\sigma(E)| < \text{const} \times E^{-\epsilon}$, $\epsilon > 0$; for "correct" boundary conditions, the spectral measure of $V(x)$ should be sufficiently close to a semiclassical estimate and, in this approximation, $V(x) < Cx^{2-\epsilon}$.

PACS numbers: 12.40.Qq