

ANALYTIC EXTRAPOLATION TECHNIQUES AND STABILITY PROBLEMS IN DISPERSION RELATION THEORY

S. CIULLI[†], C. POMPONIU* and I. SABBA-STEFANESCU**

Institute for Atomic Physics, P.O. Box 35, Bucharest, Romania

Received 11 October 1974

Abstract:

The point we try to make is that in an indirect science like elementary particle physics, it is not sufficient to have a specific description of the world brought by some happy inspiration, but rather it is necessary to optimize among large classes of (preferably among all) possible equivalent revelations. Indeed, although the leading concepts of which every description of nature makes use should bear a very close relation to the experimentally accessible data, in those situations when the basic laws are inherited from other fields, their concepts may prove to be very remote from experiment, and to “measure” them one might have to go through wildly unstable inverse problems (ill posed problems in the Hadamard sense). Moreover, the instabilities of the inverse laws become especially dangerous when the “direct laws” are too smooth, as it happens in particle physics whenever we try to cling to classical concepts (Lagrangians, interaction terms, etc.) which were purposely chosen to produce “good” classical physics laws.

To cope with this situation, one has *first* to introduce some new (experimentally or theoretically measurable) quantities, extraneous to the inherited theory, with the purpose of delimiting some compact sets inside which stable solutions of the inverse problems can be sought. Then, once the whole problem has been stabilized, there appears a *second* reason for a rational strategy of concepts and approaches, since mathematically equivalent descriptions are often rendered inequivalent by the ever existing regions where experimental knowledge lacks or is incomplete. As we try to argue in section 3, this breaking of tautologies is due to the fact that the randomness of ignorance destroys just those delicate mathematical properties (like, for instance absolute analyticity of the input) which had rendered the methods equivalent in the ideal case of total knowledge. Therefore it is of practical relevance to find among all the previous tautologic methods, that one which is least affected by our limited amount of knowledge.

This paper treats the incidence of these questions in some theoretical and phenomenological problems of particle physics, in which analytic continuation is used at least as an intermediate step.

Contents:

1. Ruling concepts, instabilities and stabilizing levers	135	4.1. Critical width ϵ_{00} of the error channel	174
1.1. Rarefaction of causal connections	135	4.2. Absolute optimum for the extrapolation to interior points	177
1.2. Inverse problems and instabilities	136	4.3. The unitary and holomorphic partial wave S -matrix, closest to the left-hand cut data	182
1.3. Indirect research	139	5. Stabilizing levers and testing of analyticity	183
1.4. Analytic extrapolation in particle physics	140	5.1. Stabilizing levers controlling the prediction power of theories	183
1.4.1. Dispersion relations	140	5.2. Critical size M_0 of the compact set and singularity hunting	184
1.4.2. Extrapolations of the Goebel—Chew—Low type	141	5.3. $M_0[h; \epsilon]$ and $\pi_0[h; \epsilon]$	186
1.4.3. Canned data	142	5.4. Computer experiment on model amplitudes	191
1.4.4. Current algebra and deep inelastic scattering	143	6. Positivity as a stabilizing lever	192
1.4.5. Dynamical schemes	143	6.1. Minimum problems and Lagrangian techniques	192
1.4.6. Resonance “hunting”	143	6.2. Best amplitudes with positivity constraints	199
1.5. Compacts and stabilizing levers	145	7. Some applications	203
1.6. Short outlook	146	7.1. Low energy $\pi\pi$ parameters and predictions of soft pion theory	203
2. Parametrizations and extrapolations to the boundary	148	7.2. πN forward scattering and crossing	204
2.1. Parametrizations versus predictions	148	7.3. Finite energy sum rules	205
2.2. Maximally converging polynomials and the optimal conformal mapping	152	7.4. Other analyticity tests	207
2.3. Stability of the optimal polynomial expansion	155	8. Outlook	207
2.4. Cut to cut extrapolations	158	Appendix A. Explicit construction of an analytic function $B(s)$, arbitrarily close to the amplitude $A(s)$ on Γ_1 , but having arbitrarily preassigned values b_i in some interior points s_i	211
2.4.1. The discrepancy method of Hamilton and Spearman	158	Appendix B. Compactness and stability	213
2.4.2. Pointwise cut to cut extrapolation and weighted dispersion integrals	158	Appendix C. The optimal mapping $w(z)$	214
2.4.3. Cut to cut extrapolation in the average	161	Appendix D. Stability of the solutions of the N/D equation	215
3. Broken tautologies and functional extremal problems	162	Appendix E. ϵ_0 and duality	216
3.1. Tautologies versus uncertainties	162	Appendix F. The Nevanlinna principle	220
3.2. Poisson weighted dispersion relations	162	References	221
3.3. Tautologies and dynamics	166		
4. Complete solution for the extrapolation to interior points	174		