Tautologies and Optimization of N/D Equations

S. Ciulli, C. Pomponiu, I. Sabba-Stefanescu, and G. Steinbrecher*

Institute for Atomic Physics, Bucharest, Romania

(Received 10 May 1972)

It is usually accepted that the solutions of the N/D equations are seriously affected by the uncertainties of the input data as well as by the lack of information about short-range forces (distant regions of the left-hand cut of the scattering amplitude). However, so far little attention seems to have been paid to a systematic investigation of the mathematical ways in which the influence of this lack of knowledge could be minimized. To this end, it appeared desirable to also include as input data the real part of the amplitude which can be found in essentially the same way as the imaginary part by analytic continuation from the crossed reactions. Then, the basic idea is to exploit the fact that different theoretical methods which are logically equivalent and yield exactly the same results for the same set of "correct" data (we call these methods "tautological") nevertheless could behave in totally different ways when faced with "wrong" (error-affected) data. "Correct" in our case would mean left-handcut limiting values absolutely consistent both with analyticity and unitarity. Hence, the problem that arises is first to find all the possible tautological integral kernels and then to choose among them that one which is most insensitive to the lack of knowledge concerning the input data. Such a kernel is shown to be obtained by replacing one of the Cauchy kernels in the integral equations by a Poisson kernel weighted with a function determined by the errors. Optimal methods of construction, not based on integral equations, are also presented.